Complete Factorization of Equations of Motion in Supersymmetric Field Theories

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We investigate bosonic sectors of supersymmetric field theories. We consider superpotentials described by one and by two real scalar fields, and we show how the equations of motion can be factorized into a family of first order Bogomol'nyi equations, so that all the topological defects are of the Bogomol'nyi-Prasad-Sommerfield type. We examine explicit models, that engender the Z_N symmetry, and we identify all the topological sectors, illustrating their integrability.

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Domain walls are defect structures that appear in diverse branches of physics. They usually live in three spatial dimensions as bidimensional objects that arise in systems described by potentials that contain at least two isolated degenerate minima. They involve energy scales as different as the ones that appear in Condensed Matter [1] and in Cosmology [2].

A lot of attention has been drawn recently to domain walls in field theories, in models that have been investigated under several distinct motivations [3–15]. A very specific motivation concerns the presence of domain walls arising in between non zero vacuum expectation values of scalar fields in supergravity [5,6]. Another line deals with the formation of defects inside domain walls [7–9]. A great deal of attention has also been drawn to $SU(N)$ supersymmetric gluodynamics, where nonperturbative effects give rise to gluino condensates that may form according to a set of N isolated degenerate chirally asymmetric vacua, from where domain walls spring interpolating between pairs of vacua [10,11].

The interest in domain walls in general widens because of the interplay between Field Theory and the low energy world volume dynamics of branes in String Theory [16–19]. In the case of intersection of defects, in particular in Ref. [13] some aspects of wall junctions have been investigated when the discrete symmetry is the Z_N symmetry, in systems described by a single complex field, with the superpotential $\varphi-\varphi^{(N+1)}/(N+1)$, $N=2,3,\dots$. The second work in Ref. [13] has shown that the tensions of the topological defects that appear in these systems can be cast to the form

$$t_{N,k}=\frac{2N}{(N+1)}\,\sin\left(\frac{k\pi}{N}\right),\quad k=1,...,\left[\frac{N}{2}\right] \tag{1}$$

where $[N/2]$ is the biggest integer not bigger than $N/2$ itself. We now know that the above result was first obtained in the first work in Ref. [3], in an investigation concerned with exact integrability, due to the presence of infinitely many conserved currents.

In the present work we examine the bosonic portion of supersymmetric theories, similar to the above models, that are of general interest to supersymmetry. In particular, we present a method that helps solving the nontrivial issue of finding Bogomol'nyi-Prasad-Sommerfield (BPS) domain walls in supersymmetric theories [11]. We start investigating systems with a single real scalar field. We write the Lagrangian density in the standard form

$$\mathcal{L}=\frac{1}{2}\,\partial_\alpha\phi\,\partial^\alpha\phi-\frac{1}{2}\,W_\phi^2 \tag{2}$$

$W=W(\phi)$ is the superpotential, and $W_\phi=dW/d\phi$. We search for defect structures, for static solutions of the equation of motion. We suppose the static solutions depend only on x, on a single spatial coordinate. The equation of motion becomes $d^2\phi/dx^2=W_\phi W_{\phi\phi}$. We examine the energy of the static configurations, and we associate with the second order ordinary differential equation of motion the two first order ordinary differential equations $d\phi/dx=\pm W_\phi$. These first order equations are Bogomol'nyi equations and their solutions are named BPS states, which saturate the lower bound in energy.

We see that solutions of the first order equations solve the second order equation of motion, and the proof follows by direct differentiation of the first order equations. We also see that one associates with the second order equation two first order equations. This fact suggests the possibility of factorizing the equation of motion into first order Bogomol'nyi equations. This is indeed the case, and we demonstrate this property by examining the ratio $R(\phi)=(d\phi/dx)/(dW/d\phi)$. We use this expression to write

$$\frac{dR(\phi)}{dx}=\left[W_\phi^2-\left(\frac{d\phi}{dx}\right)^2\right]\frac{W_{\phi\phi}}{W_\phi^2} \tag{3}$$

We notice that $dW_\phi^2/dx=2(d\phi/dx)W_\phi W_{\phi\phi}$ and that

$$\frac{d}{dx}\left(\frac{d\phi}{dx}\right)^2=2\,\frac{d\phi}{dx}\,W_\phi\,W_{\phi\phi} \tag{4}$$

where we have used the equation of motion. These results show that the quantity $S(\phi)=W_\phi^2-(d\phi/dx)^2$ does not depend on x when $\phi(x)$ solves the equation of motion. For those $\phi(x)$ we have that $\lim_{x\to-\infty}\phi(x)\to v^k$, with v^k being a vacuum state, and $\lim_{x\to-\infty}(d\phi/dx)=0$. Also, $\lim_{x\to-\infty}W_\phi=0$, since the vacuum states are extrema of the superpotential. Thus we get that $S(\phi)$ vanishes, and this allows writing $R(\phi) = \pm 1$, which gives the first order Bogomol'nyi equations. This result shows that the second order equation of motion is completely equivalent to the two first order Bogomol'nyi equations. A direct consequence of this result is that in such models all the topological solutions are of the BPS type, that is, these models do not admit the presence of non-BPS states. The first order Bogomol'nyi equations can be readily integrated, so our result offers a new way of showing exact integrability.

We illustrate our result with some examples. There are several models of systems of a single real scalar field. We work with dimensionless quantities, and we consider the superpotential

$$W^n(\phi) = \frac{1}{n+1}\,\phi^{n+1} - \frac{1}{n+3}\,\phi^{n+3}, \quad n = 0, 1, ... \tag{5}$$

Widely known examples are described by $n = 0, 1$, and identify the ϕ^4 and the ϕ^6 models, respectively. These models engender the Z_2 symmetry, and for $n = 0$ there are two asymmetric vacua, $v^1 = -1$ and $v^2 = 1$. For $n = 1, 2, ...$ there are three vacua, the former two and another one, symmetric, $v_0 = 0$, that appear with multiplicity n. For $n = 0$ the first order equations give the standard defects, $\phi(x) = \pm\tanh(x)$. These BPS solutions connect the two distinct asymmetric vacua. For $n = 1$ the solutions are such that $\phi^2(x) = (1/2)[1 \pm \tanh(x)]$. These BPS states connect the symmetric vacuum $v_0 = 0$ to the asymmetric ones, $v^{1,2} = \pm 1$. The models defined by the above superpotential are exactly solvable; the solutions can be known implicitly for $n = 2, 3, ...$ The tensions are: for $n = 0$, $t^0 = 4/3$, and for $n = 1, 2, ...$, $t^n = 2/(n+1)(n+3)$. They show that the width of the defect increases for increasing n.

The main difficulty to generalize the former result is that it is unclear how to write something like the ratio $R(\phi)$ if we deal with two or more fields. However, we have found an interesting possibility, that appears in the case of two real scalar fields, when the superpotential is harmonic on the two fields. The issue here is that for $W(\phi,\chi)$ harmonic we can work with a complex superpotential, written as $W(\varphi)$, in terms of the complex field $\varphi = \phi + i\chi$ − see Ref. [13]. In this case the ratio $R(\phi)$ generalizes to $R(\varphi)$, in the form

$$R(\varphi) = \frac{1}{\overline{W'(\varphi)}}\,\frac{d\varphi}{dx} \tag{6}$$

The superpotential $W(\varphi)$ is holomorphic, and we consider systems that are described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial_\alpha\overline{\varphi}\,\partial^\alpha\varphi - \frac{1}{2}|W'(\varphi)|^2 \tag{7}$$

where $W'(\varphi) = \partial W/\partial\varphi$. The specific form of the potential shows that the critical values of the superpotential are minima of the potential. We search for defect structures, for static solutions of the equation of motion. We suppose the static solutions depend only on x, so the equation of motion becomes

$$\frac{d^2\varphi}{dx^2} = W'(\varphi)\,\overline{W''(\varphi)} \tag{8}$$

We examine the energy of the static configurations to show that the second order ordinary differential equation is associated with a family of first order ordinary differential equations. They are

$$\frac{d\varphi}{dx} = \overline{W'(\varphi)}\,e^{-i\xi} \tag{9}$$

where ξ is some real constant. These first order equations are Bogomol'nyi equations. As usual, solutions of these first order equations also solve the second order equation of motion. The proof goes as follows: we differentiate the first order Eq. (9) to get

$$\begin{aligned}\frac{d^2\varphi}{dx^2} &= \frac{d}{dx}\overline{W'(\varphi)}\,e^{-i\xi}\\ &= \overline{W''(\varphi)}\,\frac{d\overline{\varphi}}{dx}\,e^{-i\xi} = \overline{W''(\varphi)}\,W'(\varphi)\end{aligned} \tag{10}$$

In the above equation, in the last equality we have used the first order Bogomol'nyi Eq. (9) once again.

The above models are somehow similar to the former models, described by a single real scalar field. Thus they may also lead to factorization of the equations of motion into first order Bogomol'nyi equations. We make this reasoning unambiguous by concentrating on the ratio $R(\varphi)$ defined by Eq. (6). We can write

$$\begin{aligned}\frac{dR(\varphi)}{dx} &= \left(\frac{1}{\overline{\overline{W'(\varphi)}}}\right)^2\left[\overline{W'(\varphi)}\,\frac{d^2\varphi}{dx^2} - \frac{d\varphi}{dx}\,\overline{W''(\varphi)}\,\frac{d\overline{\varphi}}{dx}\right]\\ &= \left(\frac{1}{\overline{\overline{W'(\varphi)}}}\right)^2\left[\Big|W'(\varphi)\Big|^2 - \Big|\frac{d\varphi}{dx}\Big|^2\right]\overline{W''(\varphi)}\end{aligned} \tag{11}$$

We notice that

$$\frac{d}{dx}|W'(\varphi)|^2 = \overline{W'(\varphi)}W''(\varphi)\frac{d\varphi}{dx} + W'(\varphi)\overline{W''(\varphi)}\frac{d\overline{\varphi}}{dx} \tag{12}$$

We also notice that

$$\frac{d}{dx}\left|\frac{d\varphi}{dx}\right|^2 = W'(\varphi)\overline{W''(\varphi)}\,\frac{d\overline{\varphi}}{dx} + \overline{W'(\varphi)}\,W''(\varphi)\frac{d\varphi}{dx} \tag{13}$$

where we have used the equation of motion. We use the above Eqs. (12) and (13) to see that now the quantity $S(\varphi) = |W'(\varphi)|^2 - |d\varphi/dx|^2$ is such that $dS(\varphi)/dx = 0$, that is, it does not depend on x when $\varphi(x)$ solves the equation of motion. For those $\varphi(x)$ we have that $\lim_{x\to-\infty}\varphi(x) \to v^k$, with v^k a vacuum state, and $\lim_{x\to-\infty}(d\varphi/dx) = 0$. Also, $\lim_{x\to-\infty}W'(\varphi) = 0$, because the vacuum states are critical points of the superpotential. Thus we get that $S(\varphi)$ vanishes, so $|d\varphi/dx| = |W'(\varphi)|$ and this allows writing $|R(\varphi)| = 1$. Thus we get that $R(\varphi) = e^{-i\xi}$ for some constant ξ, which leads to the first order Bogomol'nyi equations.

This is our main result, and shows that the second order equation of motion is equivalent to the first order Bogomol'nyi equations. This is valid with the boundary conditions $\lim_{x\to-\infty}\varphi(x) \to v^k$, $\lim_{x\to-\infty}(d\varphi/dx) = 0$, as required by the topological solutions. A direct consequence of this result is that in such models all the topological solutions are of the BPS type, that is, these models do not support non-BPS states.

We recall that in systems of real scalar fields, one may occasionally run into solutions that engender no topological feature. They are named nontopological solutions, and have been found for instance in Ref. [20]. We first notice that in systems of real scalar fields described by some superpotential, the nontopological solutions cannot appear as solutions of the first order Bogomol'nyi equations, because they should have zero energy, and this is the energy of the vacuum states. We add this to our former result to obtain another result, that in systems of two real scalar fields, when the superpotential is harmonic there is no room for non topological solutions.

To illustrate our result we consider the superpotential

$$W_N^n(\varphi) = \frac{1}{n+1}\varphi^{n+1} - \frac{1}{N+n+1}\varphi^{N+n+1} \tag{14}$$

It is defined by $N = 2, 3, ...$ and by $n = 0, 1, ...$ The potential is $(1/2)(\overline{\varphi}\varphi)^n\left(1-\overline{\varphi}^N\right)\left(1-\varphi^N\right)$. The models engender the Z_N symmetry, irrespective of the specific value of n. Below we obtain the tension associated with every BPS solution in a closed form.

The critical values of $W_N^n(\varphi)$ can be readily obtained. For $n = 0$ they are $v_N^k = \exp(i\xi_N^k)$, where $\xi_N^k = 2\pi(k/N)$ and $k = 1, 2, ..., N$. For $n \neq 0$, we have to add to this set of extrema the special value $v_0 = 0$, that appears with multiplicity n. Thus, while in the case $n = 0$ we get N asymmetric phases, degenerate, in the case $n \neq 0$ we have to include another phase, that breaks no symmetry of the original system. This includes the possibility of a chirally symmetric phase in $SU(N)$ supersymmetric gluodynamics.

We investigate the tension of the defect structures. Since the defects solve first order Bogomol'nyi equations, their tensions are obtained as: for $n \neq 0$, for solutions that connect the origin v_0 to any of the minima v_N^k we can write $t_{N,0}^n = |\,W_N^n(1)\,|$, that define the radial sectors. The result is, for N arbitrary, and for $n = 1, 2, ...$

$$t_{N,0}^n = \frac{N}{(n+1)(N+n+1)} \tag{15}$$

We set $N = 2$ to get $t_{2,0}^n = 2/(n+1)(n+3)$, which reproduces former result, obtained below Eq. (5).

The other sectors represent defects that connect vacua in the unit circle. Here we have to calculate the quantity $|W_N^n(v_N^N) - W_N^n(v_N^k)|$. The result can be cast to the form

$$t_{N,k}^n = 2\,t_{N,0}^n\left|\sin\left[(n+1)\frac{k\pi}{N}\right]\right| \tag{16}$$

where $k = 1, 2, ..., [N/2]$ classifies the minima on the circle that the defect connects: $k = 1$ for first neighbours, $k = 2$ for second neighbours, and so forth. We set $n = 0$ to get back to the former result, Eq. (1).

We use the superpotential (14) to write the family of Bogomol'nyi equations in the form, in terms of the real fields ϕ and χ,

$$\frac{d\phi}{dx} = \mathbf{P}_N^n\cos\xi + \mathbf{Q}_N^n\sin\xi \tag{17}$$

$$\frac{d\chi}{dx} = \mathbf{Q}_N^n\cos\xi - \mathbf{P}_N^n\sin\xi \tag{18}$$

Here we have introduced $\mathbf{P}_N^n = P_n(1-P_N) + Q_nQ_N$ and $\mathbf{Q}_N^n = Q_n(1-P_N) - P_nQ_N$, where $P_k = \mathcal{R}e\,(\phi - i\chi)^k$ and $Q_k = \mathcal{I}m\,(\phi - i\chi)^k$.

We illustrate the general situation considering particular values of N and n. The case $n = 0$ is special, because it does not admit the origin in configuration space as a vacuum state. We consider $n = 0$ and N even, $N = 2j, j = 1, 2, \ldots$. We examine the Bogomol'nyi equations for solutions that connect the vacua $(\pm1, 0)$, which are farthest neighbours in the set of $N = 2j$ minima. In this case the equations demand that $\chi = 0$, so there are only one-field solutions, obeying $d\phi/dx = \pm(1-\phi^{2j})$. In the case $j = 1$ we get to the standard tanh defects that we have already described. For $j = 2, 3, ...$ we have other solutions, that can be known implicitly. They behave as $\phi(x) = x - x^{2j+1}/(2j+1)$ in the limit $x \to 0$. They are similar to the standard defect that we have already found. However, they are thinner and then more energetic as j increases, as it can be readily verified. We use the tension result to get $t_{2j,j}^0 = 4j/(2j+1)$. This tension increases from $4/3$ for $j = 1$ up to the finite value 2 in the limit $j \to \infty$. There are other solutions, that connect other vacua in the unit circle.

In particular we consider the case $N = 2, n = 0$. The superpotential is $\phi - \phi^3/3 + \phi\chi^2$. It is instructive to see this model with another superpotential, more general, $W(\phi,\chi) = \phi - \phi^3/3 - r\phi\chi^2$. The model with $N = 2, n = 0$ is obtained for $r = -1$, the value that makes the superpotential harmonic. For $r \in (0, 1/2)$, however, the more general model maps the anysotropic XY model [1]: there are one-field solutions describing Ising walls, and two-field solutions describing Bloch walls – see the last work in Ref. [13]. For $r = -1$, for non vanishing χ one must have $\phi^2 = 1 + \chi^2/3$, so there is no two-field solution connecting the minima $(\pm1, 0)$ for $r = -1$. The lesson we learn here is that the condition $W_{\phi\phi} + W_{\chi\chi} = 0$ certainly simplifies the model, favoring its integrability.

Similar behavior is also present in the case $N = 2$, for n arbitrary. Here all the vacuum states are in the ϕ axis. The first order Bogomol'nyi equations *only* support topological solutions for $\chi = 0$, irrespective of the value of n. The Bogomol'nyi equations effectively reduce to the form $d\phi/dx = \pm\phi^n\left(1-\phi^2\right)$, which supports no nontrivial two-field solutions. They are the first order equations that appear from the superpotential of Eq. (5). The result is that for $N = 2$ and n arbitrary the systems reduce to systems of just one field. The tensions of the defects for $N=2$ are: for $n=0$, $t^0_{2,1}=4/3$, and for $n\neq 0$, $t^n_{2,0}=2/(n+1)(n+3)$, as noted below Eq.(15).

We examine $t^n_{N,k}$ to see that it vanishes for some values of N,n, and k. The condition $n=N-1$ is special, because now the models *only* support N radial sectors, that describe solutions connecting $(0,0)$ to any of the N vacua in the unit circle. For $n=N-1$ the BPS sectors effectively reduce to the sector described by just one field, and the Bogomol'nyi equations become $d\phi/dx=\pm\phi^{N-1}(1-\phi^N)$. The corresponding tensions are $t^{N-1}_{N,0}=1/2N$. The case $N=2$ reproduces the case $n=1$ of the superpotential in Eq. (5). Other values of N and n give rise to radial sectors, and to sectors that connect vacua in the unit circle. An example of this is given by $N=6$ and $n=2$. This model supports six radial sectors, with tension $t^2_{6,0}=2/9$ and six other sectors, connecting first neighbours in the unit circle, with tension $t^2_{6,1}=4/9$, twice the tension of the radial solutions.

In summary, the new idea of factorizing equations of motion into first order Bogomol'nyi equations is of direct interest to supersymmetry. It can be used in diverse applications, in particular to investigate the presence of BPS walls interpolating between distinct pairs of vacua, and exact integrability. The complete factorization of equations of motion in systems described by $W(\phi,\chi)$ harmonic, or by $W(\varphi)$ holomorphic, ensures BPS feature to domain walls that spring in such systems. Because of the associated BPS character, these domain walls partially preserve the supersymmetry. We believe it is worth examining the factorization of equations of motion into first order Bogomol'nyi equations in other models, in particular in the presence of gauge fields in higher dimensions.

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